Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Real Analysis-II

Subject Code: 4SC06REA1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date: 16/04/2019** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1	**Attempt the following questions:**	**(14)**
a)	True/False: Every continuous function is differentiable.	**(01)**
b)	State Cauchy's mean value theorem.	**(02)**
c)	Define: Lower sum of $f(x)$ w.r.t. partition	**(02)**
d)	State Darboux's theorem.	**(02)**
e)	Define: Exponential function	**(01)**
f)	Write the definition of pointwise convergence.	**(02)**
g)	Write first mean value theorem.	**(02)**
h)	State Abel's theorem.	**(02)**

Attempt any four questions from Q-2 to Q-8

Q-2	**Attempt all questions**	**(14)**
a)	Show that x^2 is integrable on any interval $[0,k]$.	**(05)**
b)	A bounded function f is R-integrable on $[a,b]$ and if derivable function $\phi(x)$ exists such that $\forall x \in [a,b]$, $\phi'(x) = f(x)$ then $\int_a^b f(x)dx = \phi(b) - \phi(a)$.	**(05)**
c)	Discuss the derivability of $f(x) = \begin{cases} 2x-3 & 0 \le x \le 2 \\ x^2 - 3 & 2 < x \le 4 \end{cases}$ at $x = 2$.	**(04)**

Q-3	**Attempt all questions**	**(14)**
a)	Show that $\frac{x-y}{1+x^2} < \tan^{-1}x - \tan^{-1}y < \frac{x-y}{1+y^2}$, if $0 < x < y$ and deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$.	**(05)**

b) If bounded function $f \in R_{[a,b]}$ then $|f| \in R_{[a,b]}$ and $\left|\int_a^b f(x)\,dx\right| = \int_a^b |f(x)|\,dx$. **(05)**

c) Evaluate: $\lim_{x\to 0}\left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right]$ **(04)**

Q-4 Attempt all questions **(14)**

a) Find the interval of convergence of the series $\frac{1}{2}x + \frac{1\cdot 3}{2\cdot 5}x^2 + \frac{1\cdot 3\cdot 5}{2\cdot 5\cdot 8}x^3 + ...$ **(05)**

b) Evaluate: $\lim_{x\to 0}\left(\frac{1}{x}\right)^{1-\cos x}$ **(05)**

c) If P^* is a refinement of a partition P then for a bounded function f then $U(P^*, f) \le U(P, f)$. **(04)**

Q-5 Attempt all questions **(14)**

a) Show that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ..., -1 \le x \le 1$. **(05)**

b) For $0 < x < \frac{\pi}{2}$, Show that $f(x) = \cos x$ is R-integrable and also find $\int_0^{\frac{\pi}{2}} \cos x\, dx$. **(05)**

c) Verify Rolle's theorem for the function $f(x) = x^3 + 3x^2 - 24x - 80$ in the interval $[-4, 5]$. **(04)**

Q-6 Attempt all questions **(14)**

a) State and prove Lagrange's mean-value theorem. **(05)**

b) Evaluate: $\lim_{x\to\infty}\left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + ... + \frac{1}{8n}\right]$ **(05)**

c) Prove that $\frac{\pi^2}{6} \le \int_0^{\pi} \frac{x}{2+\cos x}\,dx \le \frac{\pi^2}{2}$ by using general form of first mean value theorem. **(04)**

Q-7 Attempt all questions **(14)**

a) A bounded function f on $[a,b]$ is integrable if and only if for each $\varepsilon > 0$, there exist a partition P of $[a,b]$ such that $U(P, f) - L(P, f) < \varepsilon$. **(07)**

b) If $f_1 \in R[a,b]$ and $f_2 \in R[a,b]$ then prove that $f_1 \cdot f_2 \in R[a,b]$. **(05)**

c) State Taylor's theorem for power series. **(02)**

Q-8 Attempt all questions **(14)**

a) State and prove Weierstrass approximation theorem. **(07)**

b) Test for uniform convergence for the sequence $\{f_n\}$, where $f_n(x)=\dfrac{nx}{1+n^2x^2}$ for all real x. **(04)**

c) Show that the series $x^4+\dfrac{x^4}{1+x^4}+\dfrac{x^4}{(1+x^4)^2}+\dfrac{x^4}{(1+x^4)^3}+....$ is not uniformly convergent on $[0,1]$. **(03)**